

International Journal of Innovative Research in Science, Engineering and Technology

(A High Impact Factor, Monthly, Peer Reviewed Journal)

Visit: www.ijirset.com

Vol. 6, Issue 12, December 2017

Role of Bi-Lateral Laplace- Mellin Transform In Engineering

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ABSTRACT: Transformations are very important process in Mathematics and have been widely used in various fields. In this paper we show the double Bi-Lateral Laplace transform is related with Mellin transform. And we derived some properties and standard expansions with this new relation. Also used to solve one of the high frequency line equation in Network Analysis. To illustrate the advantages and use of this transformation example of differential equation has been solved at the end.

KEYWORDS: Laplace transform, Mellin transform, Bi-Lateral Laplace transform, Bi-Lateral Laplace-Mellin transform.

I. INTRODUCTION

Transform provide a powerful technique for solving initial and boundary value problems arising in applied mathematics. R. H. Mellin (1854-1933) initially gave a systematic formulation of the Mellin transform and its inverse. The interesting result of bi-lateral Laplace Mellin transform is obtained by using Ramanujan's formulae in [1-3]. Fractional Mellin transform [4,7] becomes a new way used in visual navigation [6]. Mellin transform has a better monotonicity than Fourier transform [5].

The outline of this note is as follows –

In section Ist we introduce the Bi-Lateral Laplace transform and Mellin transform. In section IInd we show the relation between transforms. In section IIIrd we discuss the general properties of Bi-Lateral Laplace-Mellin transforms. In section IVth we show the application of derivative property for the solution on one of the high-frequency line equation or radio equation using Laplace-Mellin transforms. To illustrate the advantages and use of this transformation example of differential equation has been solved at the end. In section Vth we conclude.

II. PRILIMINARIES

a) Bi-Lateral Laplace Transform

$$\bar{F}(s) = L[f(t)] = \int_{-\infty}^{\infty} e^{-st} f(t) dt \quad (2.1)$$

b) Mellin Transform

Let $f(t)$ be a function defined on the positive real axis $0 < t < \infty$. The Mellin transform 'M' is the operation mapping the function f into function F defined on the complex plane by the relation-

$$M[f(t)] = \int_0^{\infty} t^{s-1} f(t) dt \quad (2.2)$$

Where, $s \in \mathbb{C}$ is the Mellin parameter.

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III. RELATION BETWEEN TWO TRANSFORMS

In this section we see relation between Laplace and Mellin Transform by doing the change of Variable.

Combination of double Bi-Lateral Laplace transforms is denoted and defined by

$$\begin{aligned} L^2[f(x, z), s, r] &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-(sx+rz)} f(x, z) dx dz \\ &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-sx} e^{-rz} f(x, z) dx dz \\ &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-sx} (e^z)^{-r} f(x, z) dx dz \end{aligned} \tag{3.1}$$

Provided the integral exists, $r > 0$ and $s > 0$ are parameters.

Substitute $z = \log ay$ then $ay = e^z$, $dz = \frac{dy}{y}$, if $z = -\infty$ then $y = 0$ and if $z = \infty$ then $y = \infty$

$$\begin{aligned} L^2[f(x, z), s, r] &= \int_{-\infty}^{\infty} \int_0^{\infty} e^{-sx} (ay)^{-r} f(x, \log a y) dx \left(\frac{dy}{y}\right) \\ L^2[f(x, z), s, r] &= a^{-r} \int_{-\infty}^{\infty} \int_0^{\infty} e^{-sx} y^{-r-1} f(x, y) dx dy \end{aligned} \tag{3.2}$$

R.H.S. is the Bi-lateral Laplace- Mellin transform for (x, y) in the range $[-\infty, 0]$ to $[\infty, \infty]$ and denoted by $L^2[f(x, y), s, r]$

Hence $L^2[f(x, z), s, r] = L^2M[f(x, y), s, r]$

$$\text{And } L^2M[f(x, y), s, r] = a^{-r} \int_{-\infty}^{\infty} \int_0^{\infty} e^{-sx} y^{-r-1} f(x, y) dx dy \tag{3.3}$$

IV. PROPERTIES

P1] Property 1: Linearity Property:

Bi-lateral Laplace- Mellin transform is a linear operator. If a, b, c be any arbitrary constants and $f(x, y), g(x, y), h(x, y)$ are the functions, then

$$\begin{aligned} &L^2M[\alpha f(x, y) + \beta g(x, y) - \gamma h(x, y), s, r] \\ &= \alpha L^2M[f(x, y), s, r] + \beta L^2M[g(x, y), s, r] - \gamma L^2M[h(x, y), s, r] \end{aligned} \tag{4.1}$$

P2] Property 2: Change of scale property:

If α, β be any scalar and $f(x, y)$ is any function, then

$$L^2M[f(\alpha x, \beta y), s, r] = \frac{\beta^r}{\alpha \alpha^r} L^2M[f(p, q), \frac{s}{\alpha}, r] \tag{4.2}$$

By using definition of Bi-Lateral Laplace-Mellin transform,

$$L^2M[f(\alpha x, \beta y), s, r] = a^{-r} \int_{-\infty}^{\infty} \int_0^{\infty} e^{-sx} y^{-r-1} f(\alpha x, \beta y) dx dy$$

Put $\alpha x = p \implies \beta y = q$,

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As $x \rightarrow 0, p \rightarrow 0$ and as $x \rightarrow \infty, p \rightarrow \infty$

$y \rightarrow 0, q \rightarrow 0$ and as $y \rightarrow \infty, q \rightarrow \infty$

$$\begin{aligned} L^2 M [f(p, q), s, r] &= a^{-r} \int_{-\infty}^{\infty} \int_0^{\infty} e^{-s^p/\alpha} \left(\frac{q}{\beta}\right)^{-r-1} f(p, q) \frac{dp}{\alpha} \frac{dq}{\beta} \\ &= \frac{\beta^r}{\alpha a^r} \int_{-\infty}^{\infty} \int_0^{\infty} e^{-s^p/\alpha} q^{-r-1} f(p, q) dp dq \\ &= \frac{\beta^r}{\alpha a^r} L^2 M \left[f(p, q), \frac{s}{\alpha}, r \right] \end{aligned}$$

P3] Property 3: Power property:

If β be any scalar and $f(x, y)$ is any function, then

$$L^2 M [f(x, y^\beta), s, r] = \frac{a^{-r}}{\beta} L^2 M \left[f(x, t), s, \frac{r}{\beta} \right] \tag{4.3}$$

$$L^2 M [f(x, y^\beta), s, r] = a^{-r} \int_{-\infty}^{\infty} \int_0^{\infty} e^{-sx} y^{-r-1} f(x, y^\beta) dx dy$$

$$L^2 M [f(x, y^\beta), s, r] = a^{-r} \int_{-\infty}^{\infty} \int_0^{\infty} e^{-sx} y^{-r-1} f(x, y^\beta) dx dy$$

Put, $y^\beta = t \Rightarrow y = t^{\frac{1}{\beta}}, dy = \frac{1}{\beta} t^{\frac{1}{\beta}-1} dt$

As $y \rightarrow 0, t \rightarrow 0$ and as $y \rightarrow \infty, t \rightarrow \infty$

$$\begin{aligned} L^2 M [f(x, t), s, r] &= a^{-r} \int_{-\infty}^{\infty} \int_0^{\infty} e^{-sx} \left(t^{\frac{1}{\beta}}\right)^{-r-1} f(x, t) dx \frac{1}{\beta} t^{\frac{1}{\beta}-1} dt \\ &= \frac{a^{-r}}{\beta} \int_{-\infty}^{\infty} \int_0^{\infty} e^{-sx} (t)^{\left(\frac{-r}{\beta}-1\right)} f(x, t) dx dt, \\ &= \frac{a^{-r}}{\beta} L^2 M \left[f(x, t), s, \frac{r}{\beta} \right] \end{aligned}$$

P4] Property 4: First shifting property:

$$L^2 M [e^{\alpha x} f(x, y), s, r] = a^{-r} \int_{-\infty}^{\infty} \int_0^{\infty} e^{-sx} y^{-r-1} e^{\alpha x} f(x, y) dx dy$$

$$= a^{-r} \int_{-\infty}^{\infty} \int_0^{\infty} e^{-(s-\alpha)x} y^{-r-1} f(x, y) dx dy$$

$$= a^{-r} L^2 M [f(x, y), (s-\alpha), r] \tag{4.4}$$

P5] Property 5: Derivative property

$$L^2 M [f_x(x, y), s, r] = a^{-r} \int_{-\infty}^{\infty} \int_0^{\infty} e^{-sx} y^{-r-1} f_x(x, y) dx dy$$

$$= a^{-r} \int_{-\infty}^{\infty} y^{-r-1} dy \int_0^{\infty} e^{-sx} f_x(x, y) dx$$

By using integration by parts, we get

$$\begin{aligned} &= a^{-r} \int_{-\infty}^{\infty} y^{-r-1} dy \left\{ [e^{-sx} f(x, y)]_0^{\infty} - \int_0^{\infty} (-s) e^{-sx} f(x, y) dx \right\} \\ &= a^{-r} s \int_{-\infty}^{\infty} \int_0^{\infty} e^{-sx} y^{-r-1} f(x, y) dx dy - a^{-r} \int_{-\infty}^{\infty} y^{-r-1} f(0, y) dy \\ &= a^{-r} s L^2 M [f(x, y), s, r] - k \end{aligned}$$

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Where $k = a^{-r} \int_{-\infty}^{\infty} y^{-r-1} f(0, y) dy$

$$\therefore L^2 M[f_x(x, y), s, r] = a^{-r} s L^2 M[f(x, y), s, r] - k \tag{4.5}$$

Also, $L^2 M[f_{xx}(x, y), s, r] = a^{-r} \int_{-\infty}^{\infty} \int_0^{\infty} e^{-sx} y^{-r-1} f_{xx}(x, y) dx dy$

$$= a^{-r} \int_{-\infty}^{\infty} y^{-r-1} dy \int_0^{\infty} e^{-sx} f_{xx}(x, y) dx$$

By using integration by parts, we get

$$= a^{-r} \int_{-\infty}^{\infty} y^{-r-1} dy \left\{ [e^{-sx} f_x(x, y)]_0^{\infty} - \int_0^{\infty} (-s) e^{-sx} f_x(x, y) dx \right\}$$

$$= -a^{-r} \int_{-\infty}^{\infty} y^{-r-1} f_x(0, y) dy + a^{-2r} s [s L^2 M[f(x, y), s, r] - k]$$

$$= a^{-r} s^2 L^2 M[f(x, y), s, r] - sk ,$$

Where $a^{-r} \int_0^{\infty} y^{-r-1} f_x(0, y) dy = 0$, using DUIS

$$\therefore L^2 M[f_{xx}(x, y), s, r] = a^{-r} s^2 L^2 M[f(x, y), s, r] - sk \tag{4.6}$$

Similarly,

$$L^2 M[f_{xxx}(x, y), s, r] = a^{-r} s^3 L^2 M[f(x, y), s, r] - s^2 k \tag{4.7}$$

In general,

$$L^2 M[f_x^n(x, y), s, r] = a^{-r} s^n L^2 M[f(x, y), s, r] - s^{n-1} k \tag{4.8}$$

V. APPLICATIONS

In this section we apply the derivative property for the illustration of Telegraph equation and solution on one of the high-frequency line equation or Radio equation using Bi-Lateral Laplace-Mellin transforms.

1] Solving Radio Equation

Equations of transmission line are given

$$-\frac{\partial v}{\partial x} = Ri + L \frac{\partial i}{\partial t}, \quad -\frac{\partial i}{\partial x} = C \frac{\partial v}{\partial t} + G \tag{5.1}$$

$V =$ Voltage in a transmission line

$I =$ Current in a transmission line

$R =$ Resistance and $G =$ Conductance to ground

$C =$ Capacitance to ground

Elimination of 'i' or v from the two transmission line equations leads to a second order partial differential equation in i or v (Telephone equations).

By eliminating i, we get

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$$\frac{\partial^2 v}{\partial x^2} = LC \frac{\partial^2 v}{\partial t^2} + (LG + RC) \frac{\partial v}{\partial t} + RGv = 0 \quad (5.2)$$

In the case of alternating currents of high frequencies, we can neglect leakages and resistances. Thus with $R=G=0$, the above equations reduces to,

$$\frac{\partial^2 v}{\partial x^2} = LC \frac{\partial^2 v}{\partial t^2} \quad (5.3)$$

This is one of the high-frequency line equation or Radio equation similar to one dimensional wave equation. We illustrate this equation by using Bi-lateral Laplace-Mellin Transform.

The Radio equation is $\frac{\partial^2 v}{\partial x^2} = LC \frac{\partial^2 v}{\partial t^2}$

Satisfying the initial and boundary conditions

i] If $t=0$ then $v(x, 0) = 0$ for all t

ii] If $t = l$ then $v(x, l) = 0$ for all t

Solution:

$$\begin{aligned} L^2 M [v_{xx}(x, t), s, r] &= LCD_t^2 L^2 M [v(x, t), s, r] \\ \alpha^{-r} s^2 L^2 M [v(x, t), s, r] - sk &= LCD_t^2 L^2 M [v(x, t), s, r] \\ \left(D_t^2 - \frac{\alpha^{-r} s^2}{LC} \right) L^2 M [v(x, t), s, r] &= -\frac{sk}{LC} \end{aligned} \quad (5.4)$$

This is a linear differential equation with constant coefficient. Its solution is given by

$$\text{Complete Solution} = \text{C.F.} + \text{P.I.} \quad (5.5)$$

$$\text{A.E.} : \left(D_t^2 - \frac{\alpha^{-r} s^2}{LC} \right) = 0 \Rightarrow D_t = \pm \frac{s}{\sqrt{LC\alpha^r}}$$

$$\text{C.F.} = Ae^{Pt} + Be^{-Pt} \quad (5.6)$$

$$\text{Here, } p = \frac{s}{\sqrt{LC\alpha^r}}$$

$$\text{And P.I.} = \frac{1}{D_t^2 - \frac{\alpha^{-r} s^2}{LC}} \left(-\frac{sk}{LC} \right) e^0 = \frac{k}{\alpha^{-r} s}$$

$$\text{Where } k = \alpha^{-r} \int_{-\infty}^{\infty} t^{-r-1} v(0, t) dt$$

\therefore (5.5) becomes,

$$L^2 M [v(x, t), s, r] = Ae^{Pt} + Be^{-Pt} + \frac{k}{\alpha^{-r} s} \quad (5.7)$$

Applying the initial and boundary conditions,

$$A + B = \frac{-k}{\alpha^{-r} s} \text{ And } Ae^{Pl} + Be^{-Pl} = \frac{-k}{\alpha^{-r} s}$$

$$\text{We get, } A = \frac{-k}{\alpha^{-r} s} \{ (e^{-Pl} - e^{Pl})^{-1} (e^{Pl} - 1) + 1 \}$$

$$B = \frac{k}{\alpha^{-r} s} (e^{-Pl} - e^{Pl})^{-1} (e^{Pl} - 1)$$

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Putting the values in equation (5.7), we obtain

$$L^2 M[v(x, t), s, r] = \frac{-k}{a^{-r}s} \{(e^{-Pl} - e^{Pl})^{-1}(e^{Pl} - 1) + 1\} e^{Pt} + \frac{s}{a^{-r}s} (e^{-Pl} - e^{Pl})^{-1}(e^{Pl} - 1) e^{-Pt} + \frac{k}{a^{-r}s}$$

Where $p = \frac{s}{\sqrt{LCa^{-r}}}$ and $k = a^{-r} \int_{-\infty}^{\infty} t^{-r-1} v(0, t) dt$

2] Solving Telegraph equation

$$\left. \begin{aligned} \frac{\partial^2 v}{\partial x^2} &= RC \frac{\partial v}{\partial t} \\ \frac{\partial^2 i}{\partial x^2} &= RC \frac{\partial i}{\partial t} \end{aligned} \right\} \text{Telegraph equation}$$

Solving $\frac{\partial^2 v}{\partial x^2} = RC \frac{\partial v}{\partial t}$

$$\frac{\partial v}{\partial t} = \frac{1}{RC} \frac{\partial^2 v}{\partial x^2} \tag{5.8}$$

Satisfying the initial and boundary conditions,

- i] If $x=0$ then $v(t, 0) = 0$ for all x and
- ii] If $x = l$ then $v(t, l) = 0$ for x

$$L^2 M[v_t(t, x), s, r] = \frac{1}{RC} D_x^2 L^2 M[v(t, x), s, r]$$

$$D_x^2 L^2 M[v(t, x), s, r] = RCa^{-r}s L^2 M[v(t, x), s, r] - RCk, \text{ using (4.1)}$$

$$(D_x^2 - RCa^{-r}s) L^2 M[v(t, x), s, r] = -RCk \tag{5.9}$$

This is a linear differential equation with constant coefficient.

$$\text{A.E.} : (D_x^2 - RCa^{-r}s) = 0$$

$$D = \pm \sqrt{RCa^{-r}s}$$

$$\text{C. F.} = Ae^{\sqrt{RCa^{-r}s}x} + Be^{-\sqrt{RCa^{-r}s}x} \tag{5.10}$$

$$\text{And P. l.} = \frac{k}{a^{-r}s},$$

$$\text{Where } k = a^{-r} \int_{-\infty}^{\infty} x^{-r-1} v(0, x) dx$$

Hence the solution becomes,

$$L^2 M[v(t, x), s, r] = Ae^{\sqrt{RCa^{-r}s}x} + Be^{-\sqrt{RCa^{-r}s}x} + \frac{k}{a^{-r}s} \tag{5.11}$$

3] Solving telephone equation -

$$\frac{\partial^2 v}{\partial x^2} = LC \frac{\partial^2 v}{\partial t^2} + R \frac{\partial v}{\partial t}$$

Solution:

$$L^2 M[v_{xx}(x, t), s, r] = LCD_t^2 L^2 M[v(x, t), s, r] + RD_t L^2 M[v(x, t), s, r]$$

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$$a^{-r}s^2L^2M[v(x,t),s,r]-sk = LCD_t^2L^2M[v(x,t),s,r] + RD_tL^2M[v(x,t),s,r]$$

$$\left(D_t^2 + \frac{R}{LC}D_t - \frac{a^{-r}s^2}{LC}\right)L^2M[v(x,t),s,r] = -\frac{sk}{LC}$$

$$D = \frac{1}{2LC}[-R \pm \sqrt{R^2 + 4a^{-r}s^2LC}]$$

Assuming, $p = \frac{1}{2LC}[-R + \sqrt{R^2 + 4a^{-r}s^2LC}]$

$$q = \frac{1}{2LC}[-R - \sqrt{R^2 + 4a^{-r}s^2LC}]$$

$\therefore C.F. = Ae^{pt} + Be^{qt}$

And $P.I = \frac{k}{sa^{-r}}$

We get the general solution,

$$L^2M[v(x,t),s,r] = Ae^{pt} + Be^{qt} + \frac{k}{sa^{-r}} \tag{5.12}$$

VI. CONCLUSION

In this article we derived the properties for Bi-Lateral Laplace Mellin Transform. The result seems to have a potential to increase the practical utility of Bi-Lateral Laplace-Mellin transform especially in Electrical field. The Bi-Lateral Laplace-Mellin transform is the big source for Engineers to illustrate the enhanced problems.

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